Some questions on modular group algebras of finite p-groups

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Abstract: We address some questions relating the algebra structure of the group algebra kG of a finite *p*-group *G* with coefficients in the field *k* of *p* elements to the structure of the group *G* itself. A paradigmatic example of such questions is the modular isomorphism problem, which asks whether the isomorphism type of *G* can be read from kG, and to which we give positive answer provided that *G* belongs to some specific classes of *p*-groups. Another example is a question of Carlson and Kovacs about whether the group algebra of an indecomposable *p*-group (as a direct product of proper subgroups) must be indecomposable as a tensor product of proper subalgebras. We are able to show that, if such decomposition exists, none of these subalgebras can be commutative, and as a consequence we give positive answer to this question for *p*-groups that are at most 3-generated.