The lower central series of the unit group of an integral group ring

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Abstract: For a group G, denote by $\mathcal{V}(\mathbb{Z}G)$, the group of normalized units, i.e., units with augmentation one in the integral group ring $\mathbb{Z}G$. The study of $\mathcal{V}(\mathbb{Z}G)$ and its center attracts a varied set of questions and one naturally seeks the understanding of central series of $\mathcal{V}(\mathbb{Z}G)$. While the upper central series of $\mathcal{V}(\mathbb{Z}G)$ has been well explored, at least for a finite group G, apparently, not much is known about its lower central series $\{\gamma_n(\mathcal{V})\}_{n>1}$ where $\mathcal{V} := \mathcal{V}(\mathbb{Z}G)$ and

$$\gamma_1(\mathcal{V}) = \mathcal{V}, \gamma_2(\mathcal{V}) = \mathcal{V}', \gamma_i(\mathcal{V}) = [\gamma_{i-1}(\mathcal{V}), \mathcal{V}], i \geq 2$$

In this talk, I will try to draw attention towards certain fundamental problems associated to the study of the lower central series of $\mathcal{V}(\mathbb{Z}G)$ and present some recent advancements. In particular, I will present some results on the abelianisation of the $\mathcal{V}(\mathbb{Z}G)$. I would also like to discuss a natural filtration of the unit group $\mathcal{V}(\mathbb{Z}G)$ analogous to the filtration of the group G given by its dimension series, leading to results on residual nilpotence of $\mathcal{V}(\mathbb{Z}G)$.