The Procesi bundle over the Γ-fixed points of the punctual Hilbert scheme in $\mathbb{C}^2$

Raphaël Paegelow
University of Montpellier

Abstract: During my thesis, which I have done with Cédric Bonnafé and that I will defend on June 19, I have studied the fixed point locus of the punctual Hilbert scheme on $\mathbb{C}^2$ under the group action of the finite subgroups of $SL_2(\mathbb{C})$. This has been done using quiver varieties on the McKay quiver attached to the finite subgroups. Let Γ be a finite subgroup of $SL_2(\mathbb{C})$ and S_n be the symmetric group on n letters. Denote by Γ_n the wreath product of S_n with Γ. In this setting, I have shown that one can also retrieve all the projective and symplectic resolutions of the singularity $(\mathbb{C}^2)^n/\Gamma_n$, classified by Gwyn Bellamy and Alastair Craw, as irreducible components of the Γ-fixed points of k points in $\mathbb{C}^2$ where k depends on the resolution. Moreover, the indexing set of the irreducible components of the Γ-fixed point locus leads to interesting combinatorics in type A and D in terms of cores of partitions. Another direction has been to study the Procesi bundle over these irreducible components. To be more precise, I have studied the action of the group $S_n \times \Gamma$ on the fibers of the Procesi bundle over the Γ-fixed points of the Hilbert scheme of n points in $\mathbb{C}^2$. A joint work with Gwyn Bellamy using the geometry of the Procesi bundle and the geometry of the isospectral Hilbert scheme can be interpreted as a reduction to "cuspidal" fibers of the Procesi bundle. A first conjecture into the study of these cuspidal fibers seems to be using the Fock representation of the affine Kac-Moody algebra attached to Γ.