Continuum Braid group

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Abstract: In the foundational manuscript [1] Emil Artin has introduced the sequence of Braid Group B_n. B_n is a group whose elements are equivalence classes of n-braids up to isotopy. The Braid Group admits different equivalent definitions, in particular, we will introduce the Birman-Ko-Lee presentation [2] whose generators are $a_{l,m}$ (the $a_{l,m}$ braid is the elementary interchange of the l-th and the m-th strand of the braid with all the other strands held fixed). A classical result done by Lusztig [3] shows that there exists an action of the Braid Group over the Drinfel-Jimbo Quantum group (U_q^{DJ}); this action plays a central role in order to understand the structure of U_q^{DJ}. In recent years Appel, Sala and Schiffmann [3], [4] introduced a continuum analogue Quantum Group $U_q^{DJ}(X)$, that is an appropriate colimit of DJ Quantum Groups and their Cartan datum X can be thought of as a generalization of a quiver, where vertices are replaced by intervals. In order to study these continuum Quantum Groups, we define a continuum analogue of Braid Groups B_X mean by the BKL generators. We show that these groups preserve the colimit structure, we show that the Theorem of Hiwahori and Matsumoto holds [6] for the BKL presentation of B_n and it is compatible with the colimit structure.

References

[1] E. Artin, Theorie der Zöpfe, Amburg Abh. 4, (1925), 47-72

[2] J. Birman, K.H. Ko and S.J. Lee, A new approach to the word and conjugacy problem in the braid group, Adv. Math., 139 (2), (1998), 322-253

[3] G. Lusztig, "Introduction to Quantum Groups", Birkäuser, 1993

[4] A. Appel and F. Sala, Quantization of Continuum Kac Moody Algebras, Pure Appl. Math. Q. 16, (2020), 439-493

[5] A. Appel, F. Sala and O. Schiffman, Continuum Kac-Moody algebras, Moscow Mathematical Journal 22 (2022), 48pp

[6] N. Iwahori and H. Matsumoto, On some Bruhat decomposition and the structure of the Hecke rings of p-adic Chevalley groups, Inst. Hautes Etudes Sci. Publ. Math. (1965), no. 25, 5–48